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Performance and optimum geometry of spines with simultaneous heat and mass transfer

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A B S T R A C T

An analysis was carried out to study the performance of spine fins of different configurations when subjected to simultaneous heat and mass transfer mechanisms. The temperature and humidity ratio differences are the driving forces for the heat and mass transfer, respectively. Analytical solutions are obtained for the efficiency and temperature distribution over the spine surface when the surface condition is fully wet. A correction chart is developed to correct the value of the dry fin parameter if the fin surface condition is fully wet. The effect of atmospheric pressure on the spine efficiency was also studied as well as the spine optimum geometries were obtained such that a maximum amount of heat transfer rate occurs. It is shown that the closed-form solution for a dry spine case discussed in text books is a special case for the solutions presented in this paper.

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1. Introduction

Extended surfaces are used to enhance the heat transfer rate between a solid and an adjoining fluid. Extended surfaces of circular cross-section are called spines. They are widely used in many types of heat exchangers for numerous thermal engineering applications. A recent detailed review of the extended surfaces with basic heat transfer treatment was performed by Razelos [1]. However, in cooling and dehumidification processes that takes place in refrigeration and air conditioning equipment, heat and mass transfer occurs simultaneously when the spine surface temperature is lower than the dew point temperature of incoming air. In this case, the incoming moist air condenses on the spine surface. Therefore, the performance of these equipment or heat exchangers is mainly depending upon the performance (or efficiency) of spines.

Several attempts have been made to analyze the efficiency of extended surfaces with simultaneous heat and mass transfer. Kuehn et al. [2] used enthalpy difference as the driving force for the combined heat and mass transfer process to obtain an analytical expression for the overall fin efficiency. They assumed a linear relationship between the air temperature and the corresponding saturated air enthalpy. Physically, the difference in temperature and humidity ratio between the incoming air and the existing one on the fin surface are driving forces for the heat and mass transfer, respectively. It is demonstrated by Kuehn et al. [2] that the two driving forces can be combined into one which can be converted into the enthalpy difference under certain assumptions.

The overall efficiency of a fully wet straight fin is studied analytically by McQuiston [3]. He assumed that the driving force for the mass transfer, as given by the difference in the humidity ratio between the incoming air and the existing on the fin surface, is linearly related to the corresponding temperature difference. An analytical solution for the fin efficiency similar to that of the fin efficiency with only heat transfer (no mass transfer) was obtained. McQuiston demonstrated that the overall fin efficiency depends strongly on the relative humidity of the incoming air stream. Elmahdy and Biggs [4] studied numerically the overall fin efficiency of a circular fully wet fin. They considered a linear relationship between the humidity ratio of saturated air on the fin surface and its temperature, which is some what different than McQuiston's [3] model. Numerical solutions for a specific circular fin were presented. Their results indicate that the fin efficiency strongly depends on the relative humidity.

Coney et al. [5] studied heat and mass transfer mechanism in a layer adjacent to the condensate layer with heat conduction through the fin. They predicted numerically the fin surface temperature distribution, condensate film thickness and fin

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Nomenclature

a_1, a_2	parameters defined in Eqs. (12) and (15), $kg_w\,kg_a^{-1}$
A	cross-sectional or surface area, m^2
b_1, b_2	parameters defined in Eqs. (13) and (16), $kg_w\,kg_a^{-1}\,K^{-1}$
B	parameter defined in Eq. (7), °C
C	constant defined in Eq. (10), $kg_w\,kg_a^{-1}\,K^{-1}$
C_o	parameter defined in Eq. (19), $kg_w\,kg_a^{-1}\,K^{-1}$
CF	correction factor equal to $\sqrt{1+b_2B}$
c_p	specific heat of incoming moist air stream, $J\,kg^{-1}\,K^{-1}$
d	diameter, m
d^*	dimensionless spine base diameter defined in Eq. (66)
h	heat transfer coefficient on the air side, $W\,m^{-2}\,K^{-1}$
h_D	mass transfer coefficient, $kg\,m^{-2}\,s^{-1}$
i_{fg}	latent heat of evaporation of water, $J\,kg^{-1}$
k	thermal conductivity of the spine material, $W\,m^{-1}\,K^{-1}$
L	spine length, m
Le	Lewis number
m	wet spine parameter defined in Eq. (18), m^{-1}
m_o	dry spine parameter defined in Eq. (6), m^{-1}
n	spine profile exponent in Eq. (1)
N	constant in Eq. (66)
P	perimeter, m
p_{atm}	atmospheric pressure, Pa
q	heat transfer rate, W
RH	air relative humidity
T	temperature, °C
V	spine volume, m^3
x	distance from the spine tip, m
X	dimensionless distance from the spine tip

Greek symbols

η	spine efficiency
θ	dimensionless temperature defined in Eq. (8)
θ_p	dimensionless parameter defined in Eq. (27)
λ	parameter defined in Eqs. (43), (49), (55) and (61)
ω	humidity ratio of air, $kg_w\,kg_a^{-1}$
Ω	dimensionless humidity ratio defined in Eq. (9)

Subscripts

a	air
b	base
dp	dew point
fw	fully wet
max	maximum
opt	optimum
s	surface
t	tip

efficiency for a fin in a laminar humid cross-flow air arrangement. While, Chen [6] developed a two-dimensional model to analyze the fin performance with combined heat and mass transfer, he considered a second-degree polynomial relationship between the dry bulb temperature and humidity ratio for the saturated air.

Wu and Bong [7] provided an analytical solution for the efficiency of a straight fin under both fully wet and partially wet conditions by considering the temperature and humidity ratio differences as the driving forces for heat and mass transfer. They assumed the same linear relationship between the humidity ratio of saturated air on the fin surface and its temperature as that considered by Elmahdy and Biggs [4]. It is, however, important to note that the slope of this linear relationship is obtained by an iterative procedure so that the fin tip temperature can be obtained. Their result shows that there is no significant change of the fin efficiency with the relative humidity.

Kazeminejad [8] studied a rectangular fin assembly under completely wet condition. He used the concept of sensible to total heat ratio which is used in the psychrometric calculations to obtain a numerical solution for his model. His results showed that a significant decrease in fin effectiveness occurs as the amount of dehumidification increases. While Rosario and Rahman [9] investigated the radial fin assembly under completely wet operating conditions, they assumed a fixed sensible to total heat ratio to obtain their numerical solution. Their results demonstrated that there is a strong relationship between the fin efficiency and the relative humidity of incoming air.

El-Din Sala [10] investigated the performance of a partially wet fin assembly by assuming the same linear relationship between the temperature and the specific humidity that was used by McQuiston [3]. While, Laing et al. [11] introduced a polynomial variation of specific humidity with the fin surface temperature to establish a distributed simulation model for predicting steady state performance of a direct expansion air-cooling coil, they used a numerical method to calculate the partially wet and fully wet fin efficiency by taking into account the refrigerant pressure drop along the coil. In fin-and-tube heat exchangers under dehumidifying conditions, Lin et al. [12] reported a systematic study for determining the heat exchanger performance with the variation of design parameters such as inlet conditions, fin spacing and number of tube rows on the heat transfer characteristics. They estimated the fin efficiency by equivalent circular area and sectors' methods.

Kundu [13] studied analytically the performance and optimization of straight taper fins under dehumidification condition, while, Kundu and Das [14] developed a generalized analytical technique for longitudinal, annular and pin fins under dry as well as fully wet conditions. In these papers [13,14], mathematical formulation was based on the assumption of same linear relationship between the temperature and the specific humidity that was used by Wu and Bong [7]. In addition, they presented an analytical method based on Frobenius power series expansion to solve the governing differential equation. Naphon [15] numerically investigated the annular fin geometry under dry-, partially wet-, and fully wet-surface conditions. He used a third-degree polynomial correlation for the relationship between the dry bulb temperature and the humidity ratio for the saturated air.

Recently Sharqawy and Zubair [16–18] studied the efficiency and optimization of an annular fin [16], and straight fin [17] with combined heat and mass transfer. They used a modified linear relationship between the temperature and the specific humidity and were able to introduce a new fin parameter which can be used to determine the fin efficiency in fully wet and dry conditions. This modified linear relationship was tested in Ref. [18] by solving the nonlinear governing differential equation numerically and comparing the results with that obtained from the analytical solution [16]. It was found that the difference in the fin efficiency obtained analytically and numerically does not exceed more than ±3%.

It is important to note from the above studies that a linear model is generally used between the mass driving force and the temperature on the fin surface to solve the governing equation analytically. However, an iterative procedure should be carried out to get the fin tip temperature in order to establish the slope of the linear relationship. Therefore, the analytical solutions introduced by many investigators [3,4,7,10,13,14] cannot be used directly to determine the fin efficiency. We emphasize that he modified linear relationship provided by Sharqawy and Zubair [16,17] as well as the new fin parameter introduced in their work, make it easy to use the analytical solution. This result can be directly used to carry out the fin temperature distribution in addition to the overall fin efficiency for both fully wet and dry conditions.

The objective of this paper is to extend the usage of the modified linear relationship between the temperature and the specific humidity that was introduced in Sharqawy and Zubair [16,17] to solve the governing equations of spine fins having different configurations. Subsequently, to present an analytical solution for the temperature distribution and spine fin efficiency under fully wet condition, we consider the temperature and humidity ratio differences as the driving forces for heat and mass transfer mechanisms, respectively. The effect of atmospheric pressure on the spine efficiency is also investigated, in addition to the optimum dimensions of spine fins.

2. Mathematical analysis

A steady state analysis is carried out on a spine of circular cross-section and arbitrary profile when exposed to moving moist air stream, as shown in Fig. 1. In this regard, the following assumptions are made to simplify the analysis.

(a) There is a steady state heat flow.
(b) There are no heat sources or sinks in the spine.
(c) The temperatures of surrounding fluid and spine base are uniform.
(d) The spine material is homogeneous and isotropic.
(e) The moist air flow is steady and with uniform velocity.
(f) The thermal conductivity of spine, heat transfer coefficient and latent heat of condensation of water vapor are constant.
(g) The thermal resistance associated with the presence of thin water film due to condensation is small and may be neglected.
(h) The effect of air pressure drop due to air flow is neglected.

These are essentially the classical assumptions that are typically used for the analysis of extended surfaces. It may be noted that the assumption of negligible thermal resistance in the condensate film is valid as demonstrated by Coney et al. [19,20] for relative humidity and dry bulb temperature up to 90% and 35 °C, respectively. For example, during the humidification process, the thickness of condensate film is much smaller compared to the boundary layer thickness for forced convection. It is expected that the rate of condensation increases with increase of both the dry bulb temperature and the relative humidity of incoming air. However, the condensate film drains off spine surface due to gravity as well as by forced air flow.

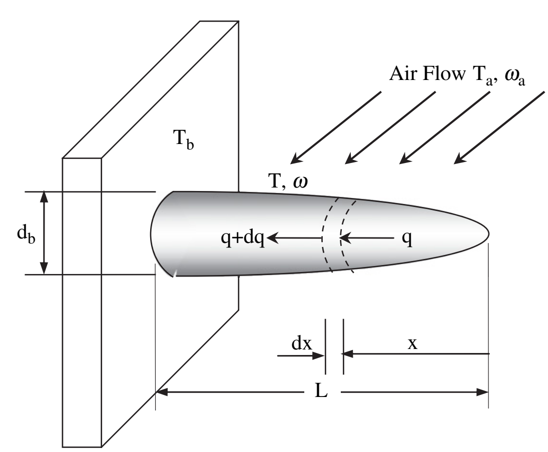

Fig. 1. Schematic of a fully wet spine fin.

Spines can be classified according to its profile as shown in Fig. 2. The spine profile is defined according to the variation of spine circular cross-section along its extended length. The diameter of the circular cross-section may vary as

$$d = d_b \left(\frac{x}{L}\right)^n \tag{1}$$

where d_b is the diameter at spine base. The spine profile exponent n, changes as follows:

(a) Spine of rectangular profile ($n = 0$) $d = d_b$
(b) Spine of triangular profile ($n = 1$) $d = d_b(x/L)$
(c) Spine of convex parabolic profile ($n = ½$) $d = d_b(x/L)^{1/2}$
(d) Spine of concave parabolic profile ($n = 2$) $d = d_b(x/L)^2$

The general differential equation that is obtained from an energy balance on an elemental volume of spine normal to the direction of heat flow, as shown in Fig. 1 is given as

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) + Ph(T_a - T) + Ph_D i_{fg}(\omega_a - \omega) = 0 \tag{2}$$

The heat transfer and mass transfer coefficients are related by the following Chilton–Colburn analogy [2]:

$$\frac{h}{h_D} = c_p \mathrm{Le}^{2/3} \tag{3}$$

Therefore, the energy balance on elemental volume results in the following dimensionless differential equation.

$$X^n \frac{d^2\theta}{dX^2} + 2nX^{n-1}\frac{d\theta}{dX} = m_o^2 L^2 \left[\theta + B\frac{\omega_a - \omega_b}{T_a - T_b}\Omega\right] \tag{4}$$

where

$$X = \frac{x}{L} \tag{5}$$

$$m_o = \sqrt{\frac{4h}{kd_b}} \tag{6}$$

$$B = \frac{i_{fg}}{c_p \mathrm{Le}^{2/3}} \tag{7}$$

$$\theta = \frac{T_a - T}{T_a - T_b} \tag{8}$$

and

$$\Omega = \frac{\omega_a - \omega}{\omega_a - \omega_b} \tag{9}$$

We emphasize that the latent heat of water evaporation, Lewis number and specific heat of air can be assumed to be constants

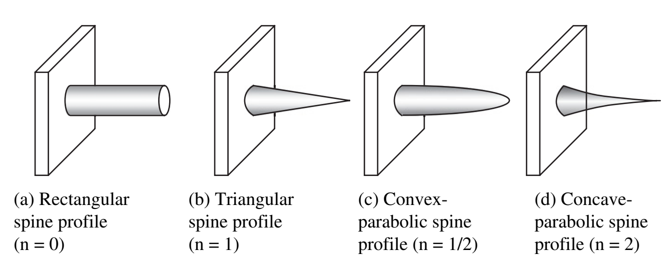

Fig. 2. Schematic of different spine fin profiles.

because the variations are not significant for a typical spine operating conditions. Thus B can be considered as a constant in Eq. (4) and has an average value of 2415 °C. Within a practical range of the air temperature and relative humidity, the variation of B value is within ±1.6% of the average value. This variation has a negligible effect on the solution.

To solve Eq. (4), an additional equation for ω is required. McQuiston [3] considered the following variation of specific humidity with temperature,

$$\omega_a - \omega = C(T_a - T) \tag{10}$$

where C is a constant. While this assumption simplifies the solution of differential equation, it is not a wide-ranging physical relationship. Eq. (4) still can be solved by making use of the fact that air near the spine surface is saturated, similar to the work of Elmahdy and Briggs [4]. However, Wu and Bong [7] used a linear relationship between ω and T over the temperature range ($T_b < T < T_t$), given by

$$\omega = a_1 + b_1 T \tag{11}$$

where

$$a_1 = \omega_b - \frac{\omega_t - \omega_b}{T_t - T_b} T_b \tag{12}$$

$$b_1 = \frac{\omega_t - \omega_b}{T_t - T_b} \tag{13}$$

While this assumption seems physically acceptable; we still don't know the temperature at the spine tip. Therefore we cannot calculate the constants a_1 and b_1 before solving the temperature distribution over the spine surface. Another improved linear relationship that is suggested by Sharqawy and Zubair [16] between ω and T over the temperature range ($T_b < T < T_{dp}$) can be expressed as

$$\omega = a_2 + b_2 T \tag{14}$$

where

$$a_2 = \omega_b - \frac{\omega_{dp} - \omega_b}{T_{dp} - T_b} T_b \tag{15}$$

$$b_2 = \frac{\omega_{dp} - \omega_b}{T_{dp} - T_b} \tag{16}$$

Here the parameters a_2 and b_2 can be calculated from the ambient air conditions and spine base temperature. There is no need for any iterative procedures to find these parameters. The spine tip temperature for fully wet condition should be lower than or equal to the dew point temperature, T_{dp} of the incoming air stream. This linear relationship was tested by Sharqawy and Zubair [18] by solving the nonlinear governing differential equation numerically and comparing the results with that obtained from the analytical solution. It was found that this assumption has a negligible effect on the results as mentioned earlier.

Substituting Eq. (14) into Eq. (4), we get

$$X^n \frac{d^2\theta}{dX^2} + 2nX^{n-1}\frac{d\theta}{dX} - m^2L^2\theta = m_o^2 L^2 B C_o \tag{17}$$

where

$$m = m_o\sqrt{1 + b_2 B} \tag{18}$$

$$C_o = \frac{\omega_a - a_2 - b_2 T_a}{T_a - T_b} \tag{19}$$

Note that Eq. (17) is a nonhomogeneous second-order differential equation with the following boundary conditions:

$$\text{At } X = 0, \quad \frac{d\theta}{dX} = 0 \tag{20}$$

$$\text{At } X = 1, \quad \theta = 1 \tag{21}$$

It is considered (refer to Eq. (20)) that heat transfer from the tip is negligible compared to that dissipated through the sides (i.e., the tip of spine is insulated).

2.1. Spine of rectangular profile (n = 0)

The solution of Eq. (17) for $n = 0$ when subjected to boundary conditions (20) and (21) gives the temperature distribution along the spine surface of rectangular profile in the form

$$\frac{\theta + \theta_P}{1 + \theta_P} = \frac{\cosh(mLX)}{\cosh(mL)} \tag{22}$$

where

$$\theta_p = \frac{BC_o}{1 + b_2 B} \tag{23}$$

The actual heat rate transferred to the spine surface, q, can be calculated from

$$q = k\frac{A_b}{L}(T_a - T_b)\frac{d\theta}{dX}\bigg|_{X=1} \tag{24}$$

where A_b is the cross-section area at the spine base. This gives

$$q = kA_b m(T_a - T_b)(1 + \theta_p)\tanh(mL) \tag{25}$$

The maximum heat transfer rate, q_{max}, that would exist if the entire spine surface was at the base temperature and saturated humidity ratio corresponding to this temperature can be calculated by the following equation:

$$q_{max} = A_s h[(T_a - T_b) + B(\omega_a - \omega_b)] \tag{26}$$

where A_s is total surface area of the spine calculated from

$$A_s = \pi d_b L \int_0^1 X^n \, dX \tag{27}$$

Substituting the total surface area into Eq. (26) will give us the maximum possible heat transfer rate which can be written as

$$q_{max} = kA_b m^2 L(T_a - T_b)(1 + \theta_p) \tag{28}$$

It is important to note that θ_p represents the mass transfer component of total heat transfer rate. Introducing the spine efficiency as the ratio of actual total heat transfer rate to maximum possible heat transfer [21,22], we get from Eqs. (25) and (28)

$$\eta = \frac{\tanh(mL)}{mL} \tag{29}$$

2.2. Spine of triangular profile (n = 1)

The solution of Eq. (17) for $n = 1$ when subjected to boundary conditions (20) and (21) gives the temperature distribution along the spine surface of triangular profile as follows:

$$\frac{\theta+\theta_p}{1+\theta_p} = X^{-1/2}\frac{I_1\left(2mL\sqrt{X}\right)}{I_1(2mL)} \tag{30}$$

The actual heat rate transferred to the spine surface, q can be calculated by using Eq. (24), this gives

$$q = kA_b m(T_a - T_b)(1+\theta_p)\frac{I_2(2mL)}{I_1(2mL)} \tag{31}$$

The maximum possible heat transfer rate, q_{max}, can be calculated by using Eq. (26) together with Eq. (27) that calculates the spine total surface area. Therefore, the maximum heat transfer rate is given by

$$q_{max} = \frac{1}{2}kA_b m^2 L(T_a - T_b)(1+\theta_p) \tag{32}$$

Using Eqs. (31) and (32), the spine efficiency can be expressed as

$$\eta = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \tag{33}$$

2.3. Spine of convex parabolic profile (n = ½)

The solution of Eq. (17) for $n = ½$ when subjected to boundary conditions (20) and (21) provides the temperature distribution along the spine surface of convex parabolic profile, in the form,

$$\frac{\theta+\theta_p}{1+\theta_p} = I_0\left(\frac{4}{3}mLX^{3/4}\right)/I_0\left(\frac{4}{3}mL\right) \tag{34}$$

The actual heat rate transferred to spine surface, q can be calculated by using Eq. (24), it gives

$$q = kA_b m(T_a - T_b)(1+\theta_p)I_1\left(\frac{4}{3}mL\right)/I_0\left(\frac{4}{3}mL\right) \tag{35}$$

As before, the maximum possible heat transfer rate, q_{max}, can be calculated by using Eq. (26) together with Eq. (27). Hence the maximum heat transfer rate can be written as

$$q_{max} = \frac{2}{3}kA_b m^2 L(T_a - T_b)(1+\theta_p) \tag{36}$$

On using Eqs. (35) and (36), spine efficiency of convex parabolic profile is given by

$$\eta = \frac{3/2}{mL}I_1\left(\frac{4}{3}mL\right)/I_0\left(\frac{4}{3}mL\right) \tag{37}$$

2.4. Spine of concave parabolic profile (n = 2)

The solution of Eq. (17) for $n = 2$ when subjected to boundary conditions (20) and (21) gives temperature distribution along the spine surface of concave parabolic profile in the form

$$\frac{\theta+\theta_p}{1+\theta_p} = X^{(3/2)(-1+\sqrt{1+(4/9)m^2L^2})} \tag{38}$$

The actual heat rate transferred to the spine surface, q can be calculated by using Eq. (24), this results in

$$q = \frac{3}{2}kA_b\frac{1}{L}(T_a - T_b)(1+\theta_p)\left(-1+\sqrt{1+\left(\frac{2}{3}mL\right)^2}\right) \tag{39}$$

The maximum possible heat transfer rate, q_{max}, can be calculated by considering Eq. (26) together with Eq. (27) that calculates the spine total surface area. Hence, the maximum heat transfer rate is

$$q_{max} = \frac{1}{3}kA_b m^2 L(T_a - T_b)(1+\theta_p) \tag{40}$$

Using Eqs. (39) and (40) and rearranging, spine efficiency of concave parabolic profile is given by

$$\eta = \frac{2}{1+\sqrt{1+((2/3)mL)^2}} \tag{41}$$

It should be noted that mathematical expressions for spine efficiency of a fully wet spine, as given in Eqs. (29), (33), (37), and (41) are the same as that for a dry spine case. The only difference is that the spine parameter m is modified by multiplying m_o by $(1+b_2B)^{1/2}$. McQuiston [3] and Wu and Bong [7] also obtained an expression for the fin efficiency under fully wet condition similar to that one under dry fin efficiency, but only for straight rectangular fins. In McQuiston's method, the parameter m is equal to $m_o(1+CB)^{1/2}$, where C is a constant defined in Eq. (10); however, in Wu's method, the parameter m is equal to $m_o(1+b_1B)^{1/2}$, where b_1 is defined in Eq. (13). It should be noted that to calculate the constant b_1 we need to know the tip condition, which can only be determined by an iterative procedure. While in the present work, the constant b_1 is replaced by b_2 (refer to Eq. (16)), which is basically the average slope of saturation line on the psychometric chart over the temperature range ($T_b < T_s < T_{dp}$). This can easily be calculated without knowing the spine tip condition.

3. Results and discussion

In order to use Eqs. (29), (33), (37), and (41) for spine fin efficiency, the modified spine parameter m should be known. Fig. 3 gives the correction factor, CF_{fw}, which when multiplied by the dry spine parameter m_o, the wet spine parameter m can be calculated. It can be seen from this figure that the correction factor is a function of air dew point, T_{dp} and spine base temperature, T_b. For a completely wet spine, air dew point should be higher than the spine base temperature by at least 7 °C. By using Figs. 3 and 4, the spine efficiency at fully wet condition can easily be established.

To illustrate the results of present work, the overall spine efficiency and temperature distribution on a spine surface have been calculated for a constant spine base temperature of 7 °C and a range of relative humidities. To facilitate a comparison between these results and those of others, the relative humidity is used here

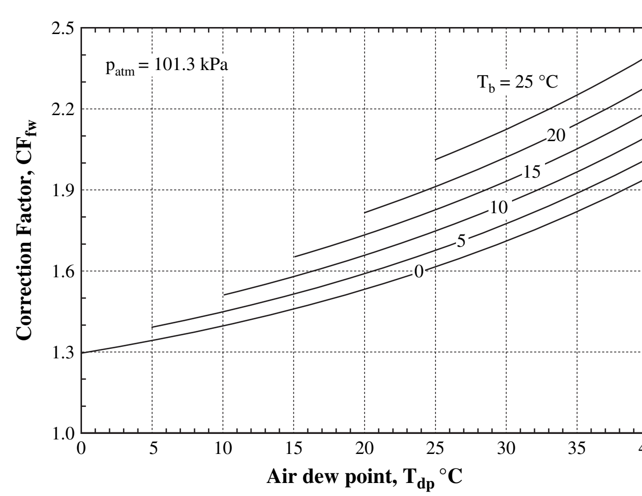

Fig. 3. Spine fin parameter correction factor.

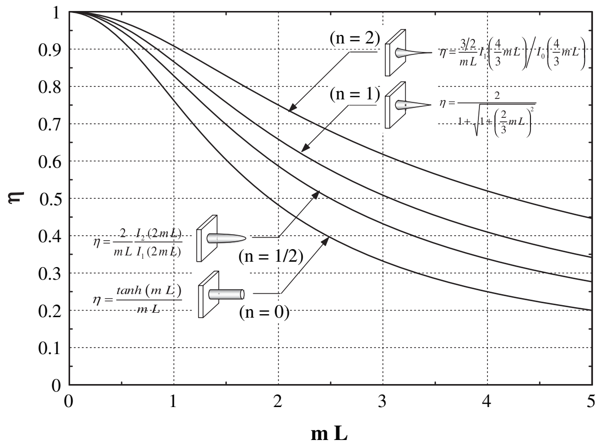

Fig. 4. Spine fin efficiency of different profiles versus *mL*.

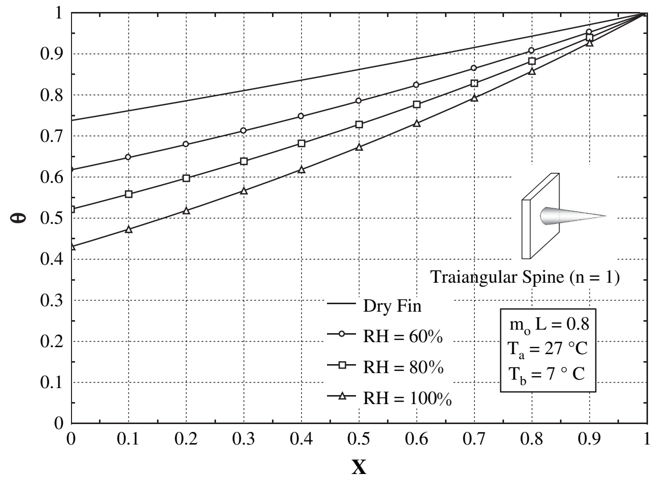

Fig. 6. Temperature distribution over a spine of triangular profile.

instead of the air humidity ratio, which is one of the important variables in the equations discussed in the previous section.

In Figs. 5–8, temperature distribution over the spine surface is plotted against the dimensionless distance from spine tip, *X*, for relative humidities, RH = 60, 80 and 100%. For these three values of relative humidities, 60, 80 and 100%, the tip temperature is found to be below the dew point of air; therefore, the spine is fully wet. It can be seen that at the same location on spine, the temperature difference between the air and spine surface is smaller for a wet spine than for a dry one. As a result, the surface temperature increases (because of heat release due to latent heat of condensation) when there is moisture condensation. It is important to note that higher the relative humidity, the higher surface temperature becomes.

For comparison purpose, the spine efficiency obtained from present work is compared with that obtained by various other methods [2,3,7] under fully wet condition. Table 1 shows comparative results of a straight rectangular fin for a relative humidity (RH) ranging from 40 to 100%. The value of fin parameter (m_oL) is chosen to be equal to 0.8. It should be noted that in using the method of McQuiston [3], spine efficiency depends strongly on RH, whereas in the methods of Kuehn et al. [2], Wu and Bong [7] and the present work, spine efficiency decreases slightly with the increase of RH. We notice that Wu and Bong's [7] approach agrees very well with the present results. It is, however, important to note that in Wu and Bong's method it is not possible to calculate the spine efficiency without knowing the tip temperature. On the other hand, there is a small difference in the spine efficiency obtained from the present (direct) analytical approach and that obtained using Wu and Bong's method. This difference depends on the values of m_oL and RH. For the entire range of RH and m_oL studied, we found that the percentage difference reaches to a maximum of 4.5% at a relative humidity value of 40% and spine (fin) parameter $m_oL = 1.2$.

The effect of variation in atmospheric pressure on the combined heat and mass transfer process has not been taken into consideration in any previous study reported in the open literature for spine fins. We know that all the psychometric properties do change with the variations in atmospheric pressure. It may happen that the heat exchanger equipments are located at high altitudes from the sea level, which means that the atmospheric pressure is different than the standard. Fig. 9 shows the efficiency of different spine fin profiles against relative humidity of air at different atmospheric pressures. In general, it can be seen that the spine efficiency increases with the increase of the atmospheric pressure. In theory, when the atmospheric pressure increases the humidity ratio of air will also increase. This increases the driving force of mass transfer process on the spine surface and hence increases the heat transfer rate due to the condensation process.

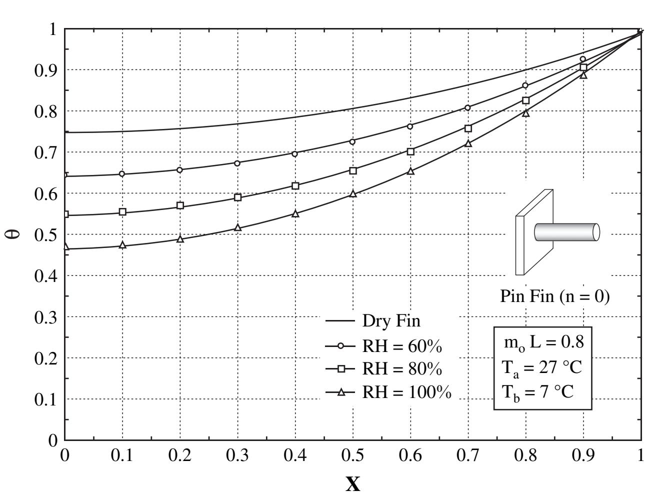

Fig. 5. Temperature distribution over a spine of rectangular profile.

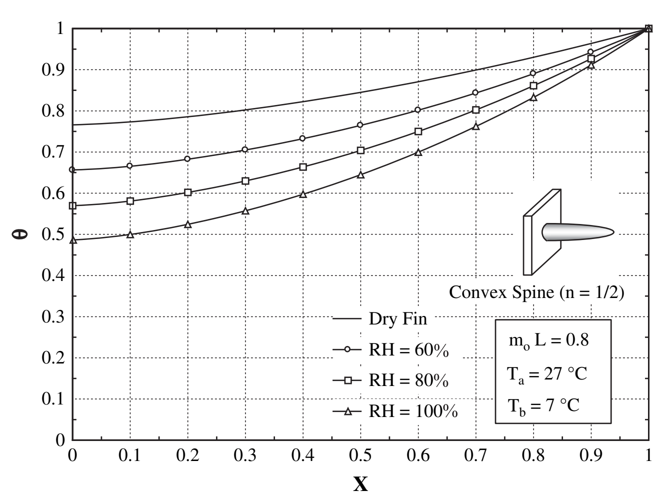

Fig. 7. Temperature distribution over a spine of convex parabolic profile.

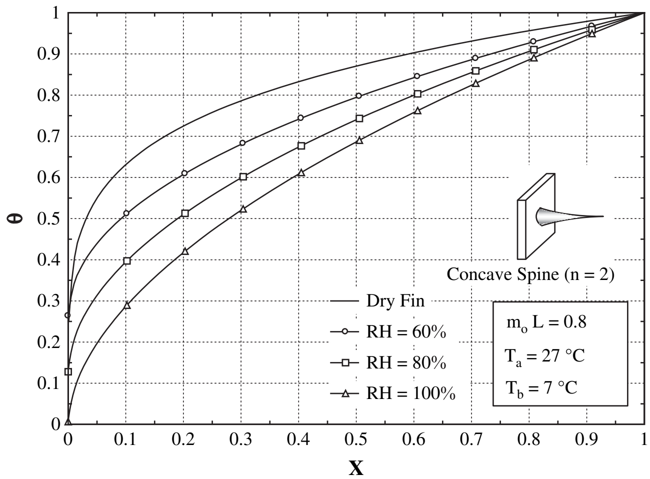

Fig. 8. Temperature distribution over a spine of concave parabolic profile.

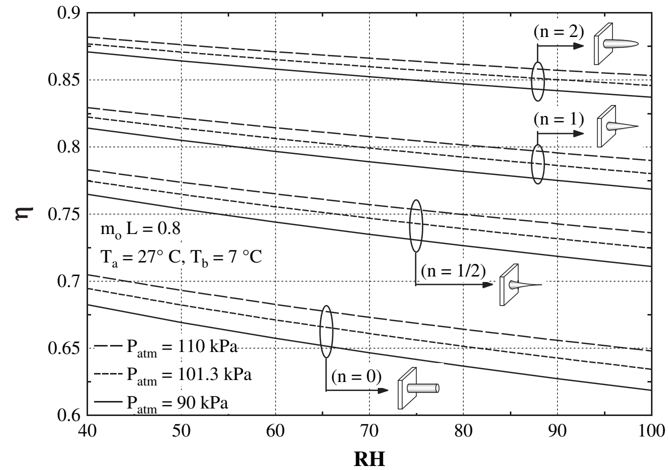

Fig. 9. Spine efficiency versus relative humidity at different atmospheric pressures.

4. Optimum spine dimensions

Spine fins shown in Fig. 2 are commonly used in many heat transfer applications where cooling and dehumidification processes occur, simultaneously. As with other fins' geometries, weight and material cost of extended surfaces are very important. Therefore, spine dimensions should be optimized so that the least amount of spine material be used to dissipate a given amount of heat flow, or alternatively that the highest dissipation rate be obtained from a given volume of spine material.

Schneider [23] provided analytical expressions for optimum fin thickness for different straight fin profiles based on dry surface conditions. An analytical expression was also derived by Sonn and Bar-Cohen [24] to give the optimum pin fin diameter for maximum heat dissipation rate. This expression was based on dry pin fin condition. Brown [25] derived an equation for a dry annular fin relating the optimum dimensions to the heat transfer rate and thermal properties of the fin and heat transfer coefficient. This relation was presented graphically in terms of suitable dimensionless parameters but again based on dry fin condition. Kundu and Das [14] recently established design curves to obtain optimum thickness of rectangular fins under fully and partially wet conditions. He used the same linear relation between humidity ratio and temperature that requires an iterative solution as that used by Wu and Bong [7]. It should, however, be noted that there is no closed-form analytical solution that can be used to determine the optimum spine dimensions when simultaneous heat and mass transfer occurs. It is, therefore, goal of this section to present optimum spine dimensions which gives the maximum heat transfer rate for longitudinal spines.

The optimum spine (fin) dimensions may be defined as those for which the spine will dissipate the maximum quantity of heat. Therefore, for any given spine profile, the optimum spine base diameter is obtained by keeping all parameters constant and considering d_b as the only independent variable in the heat transfer equation. The maximum heat transfer rate from the spine can be obtained by differentiating with respect to d_b and equating to zero.

4.1. Optimum dimensions for a spine of rectangular profile (n = 0)

The total heat transfer rate from the pin fin (spine of rectangular profile) surface when simultaneous heat and mass transfer occur is given by Eq. (25) which can be rewritten as

$$\frac{q}{\pi/4k(T_a - T_b)(1+\theta_p)} = \frac{\pi}{4}\frac{d_b^4}{V}\lambda\tanh(\lambda) \tag{42}$$

where λ is a dimensionless parameter given by

$$\lambda = \frac{4}{\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\,\frac{V}{d_b^{5/2}} \tag{43}$$

and V is the spine volume which is considered as a constant value, written as

$$V = \frac{\pi}{4}d_b^2L \tag{44}$$

Calculating the derivative$q/dd_b = 0$, we get

$$5\lambda\,\mathrm{sech}^2(\lambda) = 3\tanh(\lambda) \tag{45}$$

The positive root of Eq. (45) is

$$\lambda_{opt} = 0.9193 \tag{46}$$

Table 1
Comparison of the rectangular straight fin efficiency.

RH (%)	Kuehn et al. [2]	McQuiston [3]	Wu and Bong [7]	Present approach	% Difference with Wu and Bong [7]
40	0.660	0.790	0.670	0.695	3.6
50	0.657	0.758	0.667	0.682	2.2
60	0.653	0.727	0.663	0.671	1.2
70	0.650	0.695	0.660	0.661	0.2
80	0.647	0.663	0.657	0.651	−0.9
90	0.643	0.632	0.653	0.643	−1.6
100	0.640	0.600	0.650	0.634	−2.5

Therefore, the optimum base diameter for spine of rectangular profile for a fixed volume is

$$d_{\mathrm{b,opt}} = \left[\frac{4}{\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\ \frac{V}{0.9193}\right]^{2/5} \tag{47}$$

4.2. Optimum dimensions for a spine of triangular profile (n = 1)

The total heat transfer rate from spine of triangular profile surface when simultaneous heat and mass transfer occur is given by Eq. (31) which can be rewritten as

$$\frac{q}{(\pi/4)k(T_{\mathrm{a}}-T_{\mathrm{b}})(1+\theta_{\mathrm{p}})} = \frac{\pi}{24}\frac{d_{\mathrm{b}}^4}{V}\lambda\frac{I_2(\lambda)}{I_1(\lambda)} \tag{48}$$

where

$$\lambda = \frac{24}{\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\frac{V}{d_{\mathrm{b}}^{5/2}} \tag{49}$$

$$V = \frac{\pi}{12}d_{\mathrm{b}}^2L \tag{50}$$

and on calculating the derivative $\mathrm{d}q/\mathrm{d}d_{\mathrm{b}} = 0$, we obtain

$$6I_1(\lambda)\,I_2(\lambda)+5\lambda I_2(\lambda)[I_0(\lambda)+I_2(\lambda)] = 5\lambda I_1(\lambda)[I_1(\lambda)+I_3(\lambda)] \tag{51}$$

Finding the positive root of Eq. (51), we obtain

$$\lambda_{\mathrm{opt}} = 2.8643 \tag{52}$$

Thus, the optimum base diameter for the spine of triangular profile that has a constant profile area can be expressed as

$$d_{\mathrm{b,opt}} = \left[\frac{24}{\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\ \frac{V}{2.8643}\right]^{2/5} \tag{53}$$

4.3. Optimum dimensions for a spine of convex parabolic profile (n = ½)

The total heat transfer rate from the convex parabolic spine surface when simultaneous heat and mass transfer occur is given by Eq. (35) which can also be rewritten as

$$\frac{q}{(\pi/4)k(T_{\mathrm{a}}-T_{\mathrm{b}})(1+\theta_{\mathrm{p}})} = \frac{3\pi}{32}\frac{d_{\mathrm{b}}^4}{V}\lambda\frac{I_1(\lambda)}{I_0(\lambda)} \tag{54}$$

where

$$\lambda = \frac{32}{3\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\frac{V}{d_{\mathrm{b}}^{5/2}} \tag{55}$$

$$V = \frac{\pi}{8}d_{\mathrm{b}}^2L \tag{56}$$

and on calculating the derivative $\mathrm{d}q/\mathrm{d}d_{\mathrm{b}} = 0$, we get as

$$6I_1(\lambda)I_0(\lambda)+10\lambda I_1(\lambda) = 5\lambda I_0(\lambda)[I_0(\lambda)+I_2(\lambda)] \tag{57}$$

Finding the positive root of Eq. (57), we obtain

$$\lambda_{\mathrm{opt}} = 1.4906 \tag{58}$$

Therefore, the optimum base diameter for the spine of convex parabolic fixed profile area is

$$d_{\mathrm{b,opt}} = \left[\frac{32}{3\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\ \frac{V}{1.4906}\right]^{2/5} \tag{59}$$

4.4. Optimum dimensions for a spine of concave parabolic profile (n = 2)

The total heat transfer rate from the spine of concave parabolic surface when simultaneous heat and mass transfer occur is given by Eq. (39) which can be rewritten as

$$\frac{q}{(\pi/4)k(T_{\mathrm{a}}-T_{\mathrm{b}})(1+\theta_{\mathrm{p}})} = \frac{3\pi}{40}\frac{d_{\mathrm{b}}^4}{V}\left(-1+\sqrt{1+\lambda^2}\right) \tag{60}$$

where

$$\lambda = \frac{40}{3\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\frac{V}{d_{\mathrm{b}}^{5/2}} \tag{61}$$

$$V = \frac{\pi}{20}d_{\mathrm{b}}^2L \tag{62}$$

and on forming the derivative, $\mathrm{d}q/\mathrm{d}d_{\mathrm{b}} = 0$ we obtain

$$\lambda^2 = 5.25 \tag{63}$$

This gives

$$\lambda_{\mathrm{opt}} = 2.2913 \tag{64}$$

The optimum thickness for the concave parabolic spine of fixed profile area is therefore,

$$d_{\mathrm{b,opt}} = \left[\frac{40}{3\pi}\sqrt{\frac{4h}{k}(1+b_2B)}\ \frac{V}{2.2913}\right]^{2/5} \tag{65}$$

From Eqs. (47), (53), (59), and (65), the dimensionless optimum thickness can also be expressed as

$$d^*_{\mathrm{b,opt}} = \frac{d^5_{\mathrm{b,opt}}}{V^2(4h/k)} = (1+b_2B)\left(\frac{N}{\lambda_{\mathrm{opt}}}\right)^2 \tag{66}$$

where $(N/\lambda_{\mathrm{opt}}) = (4/0.9193\pi), (24/2.8643\pi), (32/4.4718\pi)$, and $(40/6.8739\pi)$, for rectangular, triangular, convex parabolic and concave parabolic spines, respectively. The dimensionless spine diameter given by above equation is presented as a function of mass transfer correction factor $(1+b_2B)$, which is plotted in Fig. 10. It is important to note that dry spine case results are presented by $(1+b_2B) = 1$ in this plot (that is, $B = 0$). It is clear from this figure that the optimum base diameter increases linearly with the mass transfer correction factor.

5. Concluding remarks

A closed-form analytical solution has been obtained for the efficiency as well as the total heat transfer rate of spine fins when operating under fully wet conditions. The following conclusions can be drawn from this study:

(a) A modified linear approximation model has been introduced in this work for the relation between the humidity ratio and the temperature over the spine surface.
(b) The modified linear approximation model for the relationship between humidity ratio and temperature can be used without knowing temperature at the spine tip.

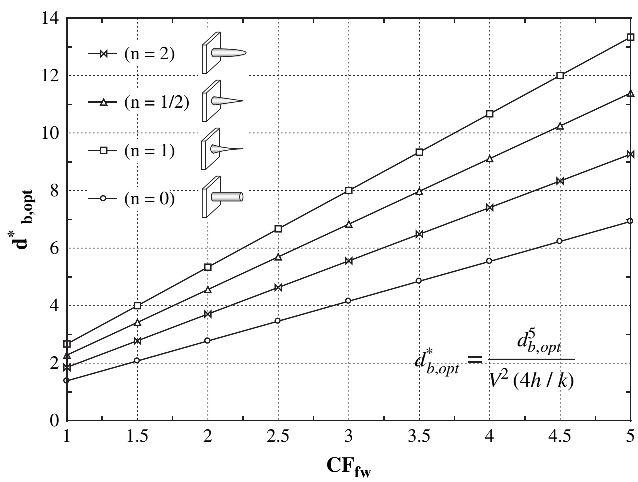

Fig. 10. Optimum dimensionless base diameter for different spine profiles.

(c) A correction chart is provided to correct the value of fin parameter in dry condition, m_o, into the fin parameter for fully wet condition.
(d) For fully wet condition, the results of present work show that overall spine efficiency is dependent on atmospheric pressure. As atmospheric pressure increases, the overall spine efficiency also increases.
(e) A dimensionless optimum spine base diameter has been introduced in this work which can be determined for both dry and fully wet conditions.

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